Numerical Simulations of Colliding Black Holes

- **General Motivation:** why is this so important now?
  - Understanding Einstein’s Theory
  - Gravitational Wave Astronomy

- **Solving Einstein’s Equations**
  - Coupled Elliptic-”Hyperbolic” system
  - Driver for Computational Physics Dev.
  - Fine interplay between analytic and numerical methods is crucial

- **Two Examples:** New physics enabled by Numerical Relativity
  - Pure Gravitational Waves
  - Black Holes
Einstein’s Equations and Gravitational Waves

Two major motivations/directions for numerical relativity

• Exploring Einstein’s General Relativity
  
  Want to develop theoretical lab to probe this fundamental theory

  – Fundamental theory of Physics (Gravity)
  – Among most complex equations of physics
    • Dozens of coupled, nonlinear hyperbolic-elliptic equations with 1000’s of terms
    • Barely have capability to solve after a century

  – Predict black holes, gravitational waves, etc, but want much more

• Exciting new field about to be born: Gravitational Wave Astronomy

  – LIGO, VIRGO, GEO, LISA, … ~ $1 Billion worldwide!
  – Fundamentally new information about Universe
  – A last major test of Einstein’s theory: do they exist?
    • Eddington: “Gravitational waves propagate at the speed of thought”
    • 1993 Nobel Prize Committee: Hulse-Taylor Pulsar (indirect evidence)
    • 20xx Nobel Committee: ??? (For actual detection…)

One century later, both of these developments happening at the same time: very exciting coincidence!
High Capacity Computing: Want to Compute
What Happens in Nature!

Teraflop Computation, AMR, Elliptic-Hyp

Perturbative

Numerical Relativity

High Capacity Computing: Want to Compute
What Happens in Nature!
Yes, but when will that ever happen?

News Flash!!
Einstein’s Equations for Numerical Relativity

- Basic object is the metric
  - Pythagorean Theorem
    - $ds^2 = dx^2 + dy^2 + dz^2$
  - Special Relativity
    - $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = -dt^2 + \sum \gamma_{ij} dx^i dx^j$
  - General Relativity
    - $ds^2 = \alpha^2 \, dt^2 + \sum \gamma_{ij} dx^i dx^j$
      - $\alpha, \beta^i$ are gauge quantities (chosen at will)
      - $\gamma_{ij}$ is the metric of the time slice, determined by Einstein’s equations
      - We speak of curved space because in a gravitational field the geometry is distorted

- Generalize Newton’s Theory
  - But much more complex
  - Eddington: “Who’s the third?”
Numerical Relativity

• Maxwell’s equations
  – Constraint Equations: \( \text{Div } B = \text{Div } E = 0 \)
  – Evolution Equations: \( \partial E/\partial t = \text{Curl } B, \partial B/\partial t = -\text{Curl } E \)
  – Gauge Condition: \( A \rightarrow A + \nabla \Lambda \)

• Einstein Eqs.: \( G_{\mu\nu}(\gamma_{ij}) = 8\pi T_{\mu\nu} \)
  – Ignore matter!
    • Shakespeare: “Much ado about nothing…”
  – Constraint Equations:
    • 4 coupled elliptic equations for initial data and beyond
    • Familiar from Newton: \( \nabla^2 \phi = 4\pi \rho \)
  – 12 evolution equations for \( \gamma_{ij}, K_{ij}(\partial \gamma_{ij}/\partial t) \)
    • Like a “wave equation” \( \partial^2 \phi/\partial t^2 - \nabla^2 \phi = \text{Source } (\phi, \phi^2, \phi') \)
    – 4 gauge conditions

• Simple Solution: \( ds^2 = -(1-2M/r)dt^2 + dr^2/(1-2M/r) + r^2 d\Omega^2 \)
  – Schwarzschild solution for spherical, static black hole
  – Want much more than this!
Einstein Equations (in axisymmetry!!!)
Important Remarks about Evolution Equations

Major research directions

• Constraints:
  – You are NOT free to impose initial data…
  – They must be solved somehow…(how?)
  – They must be preserved during evolution…(how?)

• Evolution equations:
  – Very ugly, unrecognizable form, but sort of “wave-like”
  – Many different ways to write them, all analytically equivalent:

• Physics: Interpretation very difficult since evolved quantities are coordinate and gauge dependent!

• Very difficult art: requires blend of theoretical physics, applied mathematics, computational experiment…
Computational Needs for 3D Numerical Relativity

- Get physicists + CS people together
- Initial Data: 4 coupled nonlinear elliptics
- Choose Gauge
- Evolution
  - hyperbolic evolution
  - coupled with elliptic eqs.
  - 100-200 3D arrays, TByte, Tflop crucial
- Interpret Physics

Major Goal: make this all possible for non-numerical relativist

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www.cct.lsu.edu
Scientific Computing Power Increase: Astounding!

- **Prehistoric Supercomputing ~1980**
  - 1 Professor + 1 HP calculator = 1 FLOP and 1 Byte memory

  \[ \text{Times } 10^6 \]
Grand Challenge Collaborations

Science and Eng. Go Large Scale: Needs Dwarf Capabilities

- **EU Network Astrophysics**
  - 10 EU Institutions, 3 years, €1.5M
  - Continue these problems
  - *Entire Community becoming Grid enabled*

- **NASA Neutron Star Grand Challenge**
  - 5 US Institutions, 3 years, $1.4M
  - Solve problem of colliding neutron stars (try…)

- **NSF Black Hole Grand Challenge**
  - 8 US Institutions, 5 years, $4M
  - Solve problem of colliding black holes (try…)

- **Examples of Future of Science & Engineering**
  - Require Large Scale Simulations, and collaborations
  - Bring together cross disciplinary teams
  - New Era for each field…
  - Solving problems simply beyond reach until now…

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Building Communities through Codes
Cactus Framework

Numerical Relativity

- AEI
- Southampton
- Wash U
- RIKEN
- Goddard
- Penn State
- Thessaloniki
- Tübingen
- TAC
- SISSA
- Portsmouth
- Texas, Brownsville
- Pittsburgh
- EU Astrophysics Network
- Texas, Austin
- UNAM
- LSU
- Garching
- BenchMarking
- Prototype App (Many CS Projects)

Other Applications

- Chemical Engineering (U.Kansas)
- Climate Modeling (NASA,Utrecht+)
- CFD (KISTI)
- Bio-Informatics (Chicago/Canada)
- Linear Algebra (Lebanon)
- Quantum Gravity (Hamilton)
- Plasma Physics (Princeton)
- Astrophysics (Zeus/Innsbruck)
Two Little Lessons in Black Hole Physics

• Black Hole Perturbation Theory
  – What happens when a BH is perturbed?
  – How can this be used in numerical work?
  – Going beyond linear theory through numerical relativity...

• Black Hole Horizons
  – Event vs. Apparent Horizon
  – How to find/use in numerical relativity?
  – New Tool for studying physics provided by numerical relativity...
What happens when a black hole is perturbed?

• Take existing black hole (Schwarzschild solution)
  – Analytic solution known since 1916!
  – Perturb by tossing pebble into it

• Is it stable?
  – Perturb metric tensor $g_{\mu\nu}$
    $ds^2 = -(1-2M/r)\,dt^2 + \frac{dr^2}{(1-2M/r)} + r^2\,(d\theta^2 + \sin^2\theta d\phi^2)$
  by adding $\epsilon h_{\mu\nu}(Y_{lm})$, expand to first order in $\epsilon$...
  – Discover (after 40 years study!) …
  – Scattering off potential barrier

• Find resonance frequencies of this potential barrier
  – $\psi \sim \exp(i\omega(t-x))$
  – $\omega$ resonances are complex: get damped sinusoids
  – Frequency depends on mass and spin of BH, that’s all!

• In all theoretical studies, these modes are excited
  – Collapse of matter to a BH, Distorted, Colliding BH’s...
Black Hole Normal Modes

Ringing modes: damped sinusoid $\sim e^{i\omega t}$

- Measure this, can determine mass and spin of BH that created it
  - Wavelength, damping uniquely determine spin, mass of BH…

- Questions, Questions: what happens when we go beyond linear theory?
Black Hole Horizons

- Event Horizon: outgoing light rays that *never* escape, never hit singularity
- Apparent Horizon: surface of light rays that are *instantaneously* neither expanding nor contracting

- Can measure their properties:
  - Area: \( M^2 = \frac{\text{Area}}{16\pi} \)
  - Geometry
  - Curvature
  - Embedding Shape
  - Horizon generators
  - Etc…

- Lots of physics in here!
  - Now have means to study dynamic BH’s for first time

What is shape of BH?

No embedding  Even worse!

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Event Horizons of *Dynamic* Black Holes: *Can now compute, study numerically*

“Pair of Pants” of colliding black holes. Sketched by Hawking 25 years ago, now computed quantitatively...

Color map brings out gaussian Curvature, shape can determine rotation rate!

Rotating BH Horizon: time sequence
Evolving Pure Gravitational Waves: Probing Einstein’s theory with a supercomputer

• Einstein’s equations nonlinear, so low amplitude waves just propagate away, but large amplitude waves may…
  – Scatter off themselves
  – Collapse on themselves under their own self-gravity
  – Collapse and actually form black holes

• Must use numerical relativity: Probe GR in highly nonlinear regime
  – Form BH?, Critical Phenomena in 3D?, Naked singularities?
  – … Little known about generic 3D behavior

• Take “Brill Wave” Initial data
  – \( ds^2 = \psi^4 (e^{2q}(d\rho^2+dz^2)+\rho^2d\phi^2) \)
  – \( q = A f(\rho,z,\phi) \), choose “time derivative” \( K_{ij} \) (take time symmetry for now…)
  – Large amplitude: get BH to form! Find horizon, measure its geometry, find normal modes, etc
  – Below critical value: disperses and can evolve “forever” as system returns to flat space

• We see hints of critical phenomena, known from nonlinear dynamics
Subcritical Waves: Everything radiates away...

Alcubierre, ES, et al, results

- Could not do this before 1998

- New formulations of EE’s crucial for this....

Newman-Penrose $\Psi_4$ (showing gravitational waves) with lapse $\alpha$ underneath
Collapsing Gravitational (Brill) Waves
Probing Einstein’s Theory with Supercomputers

$\Psi_4$ showing collapse to BH, Apparent Horizon with gaussian curvature
Fourier Decomposition of Waves emitted:
Full 3D simulation shows Quasinormal mode of BH after formation

$l=2, m=2$ wave extraction

Numerical waveform

QNM fit to two lowest modes for BH of appropriate mass

BH forming          Formed BH Ringing
Why are Black Holes so Difficult?

*How “Standard Numrel” breaks down.*

- **“Standard Numrel”:**
  - “ADM” formulations unstable
  - Gauge conditions unstudied…
  - Stretching of hypersurfaces leads to pathologies in metric functions

- **Why not evolve just the exterior?**
  - Horizon is Causal Boundary
  - *Physics* information only *ingoing*

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Crash!
Colliding BH Roadmap:
*A patchwork to success*

- **Merger Phase**
  - Post Newt inspiral
  - Final Plunge
  - Ringdown

- **Time Scale**
  - $t \sim 500^+ M$
  - $t \sim 100 M$
  - $t \sim 30 M$

- **Technique**
  - Post-Newtonian
  - Standard Numerical relativity:
    - Exciting new results, get farther and farther…
  - Need Post-N BH Initial data

- **Perturbative**

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Start with Traditional Approach

*Colliding Black Holes: First 3D Simulations*

- Head on, equal mass BH’s at rest: Misner data
- The classic numerical rel. problem
  - Hahn & Lindquist 1964
  - Smarr PhD thesis 1970’s
  - 2D NCSA/WashU 1990’s
- New Physics: waveforms, horizons, perturbation theory
- Simplified testbed: equal mass, axisymmetric, no spin, no momentum

Simplified testbed: equal mass, axisymmetric, no spin, no momentum
How to evolve these systems?

I. Full Numerical Relativity
• Provide Cauchy data for $g_{ij}$, $K_{ij}$
• Evolve
  – 2D
  – 3D
  – …Full Coalescence

II. Linearized Theory
• Provide Cauchy data for $\psi$, $\psi$-dot
  – Zerilli/Regge-Wheeler
  – Teukolsky
  – Easy, but “limited” validity

Need $\psi$, $\psi$-dot at $t=0$, and evolve!
Approach I: Full 3D Numerical Evolution
*Head-on, Equal Mass, BH Collisions (Misner Data) in 3D*

Event Horizon shown in green.
\( \Psi_4 \) (representing gravitational waves) shown in blue-yellow
Approach II. Comparison of Close Limit Perturbative Approach and full Numerical Relativity

- Take initial data as Cauchy problem for perturbation theory
- Evolve it using perturbative equations
- Compare to full numerical approach
- Amazing!!
- Pert theory breaks down if BH’s farther apart
- We understand Misner extremely well
- We’ll extend this later...
Grazing Collisions of 2 BHs:
Extract physics, predict rotation and mass of final BH
Initial Data for Orbits

1st Time ever: study orbital initial data via full 3D simulation

- **Cook-Baumgarte data**
  - Innermost Stable Circular Orbit
  - (ISCO, labeled by 0)
  - Other circular Orbits, too
    - Pre-ISCO 1-10
    - Focus on Pre-ISCO 3
  - Major Issue: BH data sets do not want to orbit!
    - Numerical shows inadequacy of data
    - Coalesce just after free fall time…
Evolution of Pre-ISC0 3
Are these BH’s Orbiting? No!
Coalescence Time for “Pre-ISCO Sequence”

Graph showing the relationship between the initial proper separation of AHs (M) and time (M) with different labels for various time intervals.
Coalescence Times for Pre-ISCO Sequence

expected Orbital Period

Full Numerical Coalescence Time

Med and Low Res

Newtonian Freefall

GR Head-on Collision
Analysis of Final BH for ISCO case

• Horizon Analysis
  – Study circumference geometry
  – Determine rotation
  – Compute area ---> Mass
  – QNMs excited

• Compare final BH properties to total mass, ang. mom. of spacetime
  – Compute energy, ang mom radiated
  – Find few % $M_{\text{tot}}$

<table>
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<tr>
<th>Resolution</th>
<th>0.080</th>
<th>0.060</th>
<th>0.048</th>
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<tbody>
<tr>
<td>$a/m$</td>
<td>$0.450 \pm 0.021$</td>
<td>$0.572 \pm 0.025$</td>
<td>$0.632 \pm 0.028$</td>
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<tr>
<td>$M_{\text{irr}}$</td>
<td>0.947</td>
<td>0.933</td>
<td>0.923</td>
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<td>$M_{\text{AH}}$</td>
<td>$0.973 \pm 0.003$</td>
<td>$0.978 \pm 0.005$</td>
<td>$0.980 \pm 0.006$</td>
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<tr>
<td>$J_{\text{rad}}$ (%)</td>
<td>$45.3 \pm 2.9$</td>
<td>$29.6 \pm 3.7$</td>
<td>$22.1 \pm 4.5$</td>
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<td>$E_{\text{rad}}$ (%)</td>
<td>$3.61 \pm 0.25$</td>
<td>$3.12 \pm 0.45$</td>
<td>$2.97 \pm 0.59$</td>
</tr>
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Many different BBH ISCOs!

Damour et al.

- EOB 1PN, corot (this work)
- EOB 2PN, corot (this work)
- EOB 3PN, corot (this work)
- EOB 3PN (1-2u), corot (this work)
- Non resum. 3PN, corot (Blanchet 2002)
- EOB 1PN, irrot (Buonanno & Damour 1999)
- EOB 2PN, irrot (Buonanno & Damour 1999)
- EOB 3PN, irrot (Damour et al. 2000)
- EOB 3PN (1-2u), irrot (this work)
- Non resum. 3PN, irrot (Blanchet 2002)
- IVP-punct, irrot (Baumgarte 2000)
- IVP-conf, irrot (Cook 1994, Pfeiffer et al. 2000)
- IVP-conf, S=0.08 (Pfeiffer et al. 2000)
- IVP-conf, S=0.17 (Pfeiffer et al. 2000)
- HKV, corot (Grandclement et al. 2002)
Merger Times for Simulations as of 18 Months Ago

- Only 1-2 groups could do this
- Pre-ISCO sequence w/above
- Previous uncertain data reveal transition region
- Consistent w/ Bernd et al, Meudon ISCO
- High Resolution, AMR, understanding of gauges required
Meudon Orbits: Koppitz PhD Thesis

Meudon Evolutions Inside their ISCO
Interacting with Running Jobs

Any Viz Client: LCA Vision, OpenDX

Changing any steerable parameter
- Parameters
- Physics, algorithms
- Performance

Remote Viz data

Streaming HDF5 Autodownsampling

Remote Viz data
Global Grid Testbed Collaboration

• **5 continents**
  – North America, Europe, Asia, Africa, Australia (almost had South America…)
  – Over 14 countries, including China, Japan, Singapore, S. Korea, Egypt, Australia, Canada, Germany, UK, Netherlands, Czech, Hungary, Poland, USA

• **About 70 machines, thousands of processors (~7500)**
  – Many hardware types, including PS2, IA32, IA64, MIPS, IBM Power, Alpha, Hitachi/PPC, Sparc
  – Many OSs, including Linux, Irix, AIX, OSF, True64, Solaris, Hitachi

• **Many organizations**
  – DOE, NSF, MPG, Universities, Vendors

• **And yet: all run same Grid infrastructure, and many can be used for applications…**
The Grid Testbed and Status

See http://sc2002.aei.mpg.de
Recent Progress in Community: Exploring Orbits!

- Frans Pretorius (PRL 95:121101, 2005 and gr-qc/0602115) uses a generalized harmonic formulation, collapsing scalar field initial data, compactified coordinate system placing the outer boundaries at spatial infinity, AMR, moving excision.

- Diener et. al (gr-qc/0512108, PRL) uses the BSSN formulation, puncture initial data, fixed excision, adaptive comoving coordinates, a slightly modified $\Gamma$-driver shift and fixed mesh refinement.

- Baker et. al (gr-qc/0511103 and gr-qc/0602026) and Campanelli et. al (gr-qc/0511048 and PRD 73:061501, 2006) independently came up with very similar schemes to evolve moving punctures using no excision, BSSN and slightly modified $1+\log$ and $\Gamma$-driver gauge conditions. Baker et. al uses AMR (Paramesh).
Evolutions of collapsing scalar field data with different initial boosts of the scalar field distributions. The orbital characteristics and waveforms are extremely sensitive to the initial data boost.
“D3.0” with an initial proper separation of \( L=9.32 \). Estimated common AH time is \( T=125M \).

Remarkable agreement between the plunge part of the QC0 and D3.0 waveforms.
LSU/AEI Results

- Orbital dynamics depend sensitively on gauge and resolution
- It is possible to reconcile apparently different results at high enough resolution
- Resolved discrepancy with results of Bruegmann, LSU/AEI
LSU/AEI Results

Duration of last orbit before AH formation is 59M. Common apparent horizon formation at T=124M.

Angular velocity at merger agrees with half quasinormal mode frequency of the final BH: period of 15.9M corresponds to $\omega=0.395$ (pointed out by Steve Detweiler).
Goddard Results

• Surprisingly good overlap between late time orbital dynamics as well as the shifted waveforms.
The Future

• Gravitational wave astronomy almost here: must be able to solve Einstein’s equations to understand the new observations
• Numerical treatment will be a most important tool for studying Einstein’s theory (and other theories…)
  – Need new computational and numerical algorithms
    • Computational Science, Grids (www.gridforum.org)
  – Synergies with many analytic methods
• New Physics Now, much more coming...
  – Collapse of 3D gravitational waves to BH
  – Patching together 3D simulations for Gravitational Wave Astronomy
  – Horizon dynamics, physics
  – 3D Neutron Star simulations, relativistic astrophysics are just now possible: bright future…
• Numerical Relativity in early maturity phase:
  – Simple ideas, coupled with advanced computational infrastructure, still have large impact
  – jean-luc.aei.mpg.de for more info…